Exercise 1

Find the linearization $L(x)$ of the function at a.

$$f(x) = x^3 - x^2 + 3, \quad a = -2$$

Solution

Start by finding the corresponding y-value to $x = -2$.

$$f(-2) = (-2)^3 - (-2)^2 + 3 = -9$$

Then find the slope of the tangent line to the function at $x = 2$ by computing $f'(x)$,

$$f'(x) = \frac{d}{dx}(x^3 - x^2 + 3)$$

$$= 3x^2 - 2x,$$

and plugging in $x = -2$.

$$f'(-2) = 3(-2)^2 - 2(-2) = 16$$

Now use the point-slope formula to obtain the equation of the line going through $(-2, -9)$ with slope 16.

$$y - f(-2) = f'(-2)(x - (-2))$$

$$y - (-9) = 16(x + 2)$$

$$y + 9 = 16x + 32$$

$$y = 16x + 23$$

Therefore, the linearization of the function $f(x)$ at $a = -2$ is

$$L(x) = 16x + 23.$$

Below is a plot of the function and the linearization at $a = -2$ versus x.